# Shot-noise in Transport and Beam Experiments

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Consider two Fermi gases with the same *average* currents: a *transport gas*, as in solid-state experiments where the chemical potentials of terminal 1 is $\mu + eV$ and of terminal 2 and 3 is $\mu$, and a *beam*, i.e., electrons entering only from terminal 1 having energies between $\mu$ and $\mu + eV$. By expressing the current noise as a sum over single-particle transitions we show that the temporal current *fluctuations* are very different: The beam is noisier due to allowed single-particle transitions into empty states below $\mu$. Surprisingly, the correlations between terminals 2 and 3 are the same.

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The subject of quantum shot noise[1,2,3] has recently been of major interest, for example, due to the possibility to observe different quasi-particle charges of the carriers[4]. Attempts to examine analogies with Hanbury-Brown and Twiss[5,6,7] correlations deserve particular attention. In 1918 Schottky[8] observed that one contribution (called shot-noise) to the noise in currents flowing in vacuum tubes was due to the discreteness of the electrons. Presently, most experiments on electronic noise (an exception is, e.g., Ref. 7) are performed in a degenerate Fermi gas and not in vacuum beams. Despite that, they are often analyzed in a similar fashion to vacuum beams[5,6,9]. Below it is shown that this point of view is not justified since the temporal noise correlators in a given terminal are substantially different in beams and degenerate Fermi systems (surprisingly the correlations between different terminals turn out to be same). To show this, we shall apply our approach[10] of viewing noise as the radiation (or excitations of a detector[11]) produced by the current fluctuations.

We consider current fluctuations for two types of Fermi gases in a ballistic conductor which consists of three single-channel arms connected to an elastic scatterer (fig.1), assuming zero temperature and non-interacting electrons. The scattering state with energy $\epsilon_n = k^2/2m$, corresponding to a wave that is incoming on arm $\alpha$, partially reflected back into it and partially transmitted into the other arms, is: $\varphi_n(x_\beta) = L^{-1/2}[\delta_{\alpha\beta}e^{-ikx_\beta} + s_{\beta\alpha}(k)e^{ikx_\beta}]$. Here $n \equiv (\alpha, k)$ with $k > 0$, $s_{\alpha\beta}$ is the scattering matrix, $\alpha, \beta = 1, 2, 3$, $L$ is a normalization length, $m$ the electron mass and $x_\beta$ the distance of a point on arm $\beta$ from the scatterer. To specify that a state $\varphi_n$ comes from terminal $\alpha$, we shall write $n \in \alpha$.

We compare the current fluctuations in two many-body states (fig.2), the *transport gas*:

$$|transport\rangle \equiv \prod_{\substack{n\in 1;\ \mu\le\epsilon_n\le\mu+eV \\ n\in 1,2,3;\ \epsilon_n\le\mu}} \hat{a}_n^\dagger |vacuum\rangle, \tag{1}$$

and the *beam*:

$$|beam\rangle \equiv \prod_{n\in 1;\ \mu\le\epsilon_n\le\mu+eV} \hat{a}_n^\dagger |vacuum\rangle. \tag{2}$$

where $\hat{a}_n$ and $\hat{a}_n^\dagger$ are the annihilation and creation operators of the $\varphi$'s. In the transport gas all the $\varphi_n$'s are occupied up to an energy $\mu$ if $n \in 2, 3$ and up to $\mu + eV$ if $n \in 1$. In the beam the only occupied states are all those coming in on arm 1, which are in the energy range $[\mu, \mu + eV]$. It is assumed that $eV \ll \mu$.

The current operator on the arm $\beta$ is $\hat{j}(x_\beta) = -(ie/2m)\sum_{nn'} \hat{a}_n^\dagger \hat{a}_{n'} \varphi_n^* \nabla_\beta \varphi_{n'} + h.c.$. We assume that the *measured* current is the average

$$\hat{J}_\beta \equiv \frac{1}{L_0}\int_{L_0} dx_\beta \hat{j}(x_\beta) \tag{3}$$

over a segment $L_0$ far away from the scatterer (fig.1) which satisfies: $L_0 k_F \gg 1$ and $\omega L_0 m/k_F \ll 1$, where $k_F \equiv \sqrt{2m\mu}$, and $\omega$ is the frequency of the measured noise which is assumed to satisfy $\omega \ll \mu$. These conditions ensure that the current correlations are independent of the length and position of the segment $L_0$, and thus will have no spatial dependence, which is not addressed in experiments.

We consider correlators of the current fluctuations in the frequency domain:

$$S_{\alpha\beta}(\omega) \equiv \int_{-\infty}^{\infty} e^{i\omega t} \langle i|\hat{J}_\alpha(0)\hat{J}_\beta(t)|i\rangle dt, \tag{4}$$

where $\hat{J}_\alpha(t) = e^{iHt}\hat{J}_\alpha e^{-iHt}$ is the Heisenberg representation of $\hat{J}_\alpha$ and $|i\rangle = |beam\rangle, |transport\rangle$. There is an alternative definition as a Fourier transform of the symmetrized correlator $(1/2)\langle i|\hat{J}_\alpha(0)\hat{J}_\beta(t) + \hat{J}_\beta(t)\hat{J}_\alpha(0)|i\rangle$. We use the non-symmetrized version since, following ref. 11, we showed[10] that at least for $\alpha = \beta$ and for some types of noise detection, it is Eq.(4) which gives the measured noise if the detector is cold enough.

Following the ideas in neutron-scattering theory introduced by Van-Hove[12] we insert a complete set of eigenstates into Eq.(4) and get after a short manipulation:

$$S_{\alpha\beta}(\omega) = 2\pi \sum_f \langle i|\hat{J}_\alpha(0)|f\rangle\langle f|\hat{J}_\beta(0)|i\rangle \delta(E_i - E_f - \omega), \tag{5}$$

where $E_i$ and $E_f$ are the energies of $|i\rangle$ and $|f\rangle$. The non-diagonal element $\langle i|\hat{J}_\alpha(0)|f\rangle$ is nonzero only if $|f\rangle$ differs

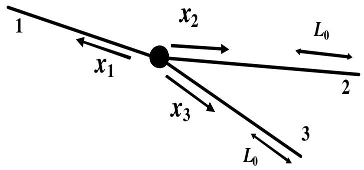


FIG. 1: Three leads connected to an elastic scatterer

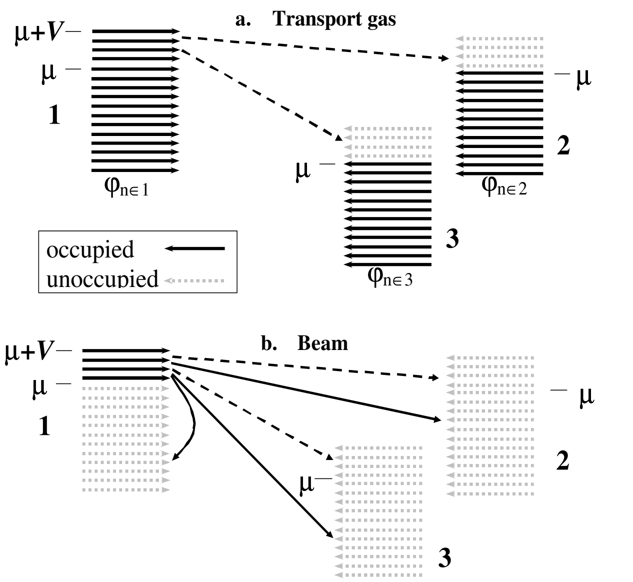


FIG. 2: The occupations and possible transitions for $\omega > 0$. Short horizontal arrows represent scattering states.

from $|i\rangle$ by moving one particle from an occupied state, $\varphi_n$, to a previously unoccupied state, $\varphi_{n'}$, i.e., $|f\rangle$ is of the form $\hat{a}^\dagger_{n'}\hat{a}_n|i\rangle$ (up to a fermionic factor $c = \pm 1$, that will play no role below.) The term with the diagonal element $\langle i|\hat{J}_\alpha(0)|i\rangle$ which is the average current, $I_\alpha(V)$, on arm $\alpha$, yields a term $\sim \delta(\omega)$. In what follows we consider only $\omega \neq 0$ and therefore neglect this term. In experiments the integration in Eq.(4) is limited by the sampling time of the experiment, $T_s$, and as a result $\delta(\omega)$ is smoothed into a peak with a width of $\simeq 1/T_s$ which means that the condition $\omega \neq 0$ actually is $\omega T_s \gg 1$. We therefore have:

$$S_{\alpha\beta}(\omega) = 2\pi \sum_{nn'} J_{\alpha,nn'} J^*_{\beta,nn'} \delta(\epsilon_n - \epsilon_{n'} - \omega), \qquad (6)$$

where $J_{\alpha,nn'} \equiv \langle i|\hat{J}_\alpha(0)\hat{a}^\dagger_{n'}\hat{a}_n|i\rangle$, and where now the summation over $n$ and $n'$ is over all *single-particle* states $\varphi_n$ and $\varphi_{n'}$ which are occupied and unoccupied, respectively, in $|i\rangle$. The auto-correlator is

$$S_{\alpha\alpha}(\omega) = 2\pi \sum_{nn'} |J_{\alpha,nn'}|^2 \delta(\epsilon_n - \epsilon_{n'} - \omega). \qquad (7)$$

When the system is coupled to a measuring device (e.g., some circuit or an electro-magnetic field) through a small term linear in $\hat{J}_\alpha$, $S_{\alpha\alpha}(\omega)$ is a sum over *single-particle transitions*, the probability of each given by the Fermi golden rule, between an initial $\varphi_n$ and a final $\varphi_{n'}$. (The cross-correlator, Eq.(6) for $\alpha \neq \beta$, should not be viewed similarly since then $J_{\alpha,nn'}J^*_{\beta,nn'}$ is not a transition amplitude squared). Via these transitions energy is transferred between the system and the measuring device: terms with $\epsilon_n > \epsilon_{n'}$ (one particle goes down in energy) describe transitions in which an energy of $\omega = \epsilon_n - \epsilon_{n'} > 0$ is transferred from the system to the measuring device, while terms with $\epsilon_{n'} > \epsilon_n$ (one particle goes up) describe transitions in which an energy of $-\omega = \epsilon_{n'} - \epsilon_n > 0$ is transferred from the measuring device to the system. When $\omega > 0$, only the first type of terms will remain and $S_{\alpha\alpha}(\omega)$ will be the emission spectrum while $S_{\alpha\alpha}(-\omega)$ is the absorption spectrum. Thus we conclude that when $\alpha = \beta$ there will be emission of noise at frequency $\omega$ *if and only if* there exist occupied and unoccupied states in $|i\rangle$, $\varphi_n$ and $\varphi_{n'}$, with $|J_{\alpha,nn'}|^2 \neq 0$ and $\omega = \epsilon_n - \epsilon_{n'} > 0$. For $\alpha \neq \beta$, this is not necessarily so, since the terms in Eq.(6) are complex and may cancel.

Now let us compare the current and its fluctuations in the transport gas and the beam, considering the arms 2 and 3. The average currents in both systems are defined only by states in the energy window $[\mu, \mu + eV]$ and are the same: for $\beta = 2, 3$ one finds $I_\beta(V) = e^2 T_\beta V/(2\pi)$ both for $|i\rangle = |beam\rangle$ and $|i\rangle = |transport\rangle$, where $T_\beta \equiv |s_{\beta 1}|^2$. (For simplicity we neglected the energy dependence of the transmission). By calculating the emission spectrum ($\omega > 0$) we now show that the current fluctuations may differ. Rewriting Eqs.(6) and (7) for $\omega > 0$, taking into account the energy conservation and the different occupations in the states Eqs.(1) and (2), one has for the transport gas:

$$S^{tr}_{\alpha\beta}(\omega) = 2\pi \sum_{\substack{n\in 1, n'\in 2,3 \\ \mu<\epsilon_{n'}<\epsilon_n<\mu+eV}} J_{\alpha,nn'} J^*_{\beta,nn'} \delta(\epsilon_n - \epsilon_{n'} - \omega) \qquad (8)$$

and for the beam:

$$S^b_{\alpha\beta}(\omega) = S^{tr}_{\alpha\beta}(\omega) + \sum_{n\in 1;\ \mu<\epsilon_n<\mu+\min(\omega,eV)} S^{(n)}_{\alpha\beta}(\omega). \qquad (9)$$

Here $S^{(n)}_{\alpha\beta}(\omega)$ corresponds to the correlator in the state $\hat{a}^\dagger_n|vacuum\rangle$, which is a beam with a *single* particle, in a state $\varphi_n$:

$$S^{(n)}_{\alpha\beta}(\omega) = 2\pi \sum_{n'\in 1,2,3} J_{\alpha,nn'} J^*_{\beta,nn'} \delta(\epsilon_n - \epsilon_{n'} - \omega). \qquad (10)$$

$S^{tr}_{\alpha\beta}(\omega)$ contains transition amplitudes between occupied states in the energy window $[\mu, \mu + eV]$ to lower empty states inside the same energy window. These transitions, shown by dashed arrows in figs. 2a, and 2b, are possible both in the transport gas and the beam and therefore $S^{tr}_{\alpha\beta}(\omega)$ appears also in Eq.(9). Contrary to the first one, the second term in Eq.(9) contains transition amplitudes between occupied states in the energy window $[\mu, \mu + eV]$ to empty states below $\mu$ (long solid arrows in fig. 2b), transitions which are allowed only in the beam. Writing

this term as a sum of single-particle correlators was possible since in the beam all the levels below $\mu$ are empty so the sum runs over all possible values of $n'$ with a given energy, unlike in Eq.(8) for the transport gas where $n' \notin 1$.

Now, the current matrix element is given by:

$$\langle i|\hat{j}_\alpha\ \hat{a}^\dagger_{n'}\hat{a}_n|i\rangle = (c/2)\Big[(k'+k)e^{i(k-k')x_\alpha}s_{\alpha 1}(k)^*\times \\ s_{\alpha\gamma'}(k') + (k'-k)s_{\alpha 1}(k)^*\delta_{\alpha\gamma'}e^{-i(k+k')x_\alpha}\Big] \quad (11)$$

for $n=(k,1)$ occupied and $n'=(k',\gamma')$ empty. $c=\pm 1$, as above. Performing the average as defined in Eq.(3) and using the conditions for $L_0$, we obtain

$$J_{\alpha,nn'} = (c/2)(k+k')s_{\alpha 1}(k)^* s_{\alpha\gamma'}(k'). \quad (12)$$

This matrix element has no spatial dependence because the fast oscillating term in Eq.(11) vanished while the slow oscillating one is constant within $L_0$. Inserting Eq.(12) into Eq.(6), transforming the sums over $k$ and $k'$ into integrals, integrating using the condition $\omega, eV \ll \mu$, using the unitarity of the scattering matrix, $\sum_\gamma s_{\alpha\gamma}s^*_{\beta\gamma} = \delta_{\alpha\beta}$, one gets[1] for $\alpha,\beta = 2,3$, and $\omega > 0$:

$$S^{tr}_{\alpha\beta}(\omega) = \frac{e^2}{2\pi}T_\alpha(\delta_{\alpha\beta} - T_\beta)(eV-\omega)\theta(eV-\omega)\ \ , \quad (13)$$

where $\theta$ is the Heaviside step-function. Similarly, using Eq.(12) in Eq.(10) one gets for the single-particle correlator (see discussion below), for $\alpha,\beta = 2,3$:

$$S^{(n)}_{\alpha\beta}(\omega) = \delta_{\alpha\beta}eI_{n,\alpha}, \quad (14)$$

where $I_{n,\alpha} = eT_\alpha(k/m)(1/L)$ is the average current on arm $\alpha = 2,3$ of a single particle in the state $n=(k,1)$.

Substituting Eq.(14) in Eq.(9), one gets for $\alpha,\beta = 2,3$:

$$S^b_{\alpha\beta}(\omega) = S^{tr}_{\alpha\beta}(\omega) + \delta_{\alpha\beta}eI_\alpha(\omega), \quad (15)$$

where $I_\alpha(\omega)$ is the average current of the electrons in the energy window $[\mu, \mu+\min(\omega, eV)]$:

$$I_\alpha(\omega) \equiv \sum_{n\in 1;\ \mu<\epsilon_n<\mu+\min(\omega,eV)} I_{n,\alpha} = I_\alpha(V)\frac{\min(\omega,eV)}{eV}. \quad (16)$$

Eq.(15) is our main result and it demonstrates that although the *average* currents in arms 2 and 3 in the beam and the transport gas are the same, the current *fluctuations* in these arms generally differ. The beam has much more noise: e.g., for $\alpha = 2,3$ the auto-correlation spectra of the transport state $S^{tr}_{\alpha\alpha}(\omega)$ has an upper cutoff at $\omega = eV$, but the auto-correlation spectra of the beam $S^b_{\alpha\alpha}(\omega)$ has no such cutoff. The spectra $S^b_{\alpha\alpha}(\omega)$ at $\omega > eV$ is given by the extra second term in in Eq.(15). Interestingly, this term is identical to the result for a beam of uncorrelated (Poissonian) classical particles[13] which carries an average current given by Eq.(16). Surprisingly, the cross-correlation, $S_{23}(\omega)$ is identical in the beam and the transport gas, since this term vanishes for $\alpha \neq \beta$.

According to Eq.(15) and Eq.(16) $S^b_{\alpha\alpha}(\omega)$ and $S^{tr}_{\alpha\alpha}(\omega)$ start to differ substantially for $\omega$ of order $eV$. The measurement in Ref. 6 is consistent with Eq.(13) but since it is performed at $\omega \ll eV$, it can not distinguish between $S^{tr}_{\alpha\alpha}(\omega)$ and $S^b_{\alpha\alpha}(\omega)$. In Ref. 5 (see particularly Fig.3) it is claimed that the cross-correlation are measured in the time domain. The function that was obtained via this measurement has characteristic time-scale of $\sim 100ns$ which, in the frequency domain, corresponds to $10MHz$. However, in both Eq.(15) and Eq.(16) the only characteristic frequency scale is of the order of $eV \approx 10^5 MHz$ (estimated for $\sim 30nA$ and transmission of order 1), that is many orders of magnitude larger. Thus, the results in Ref. 5 are not consistent with ours.

The simple case of a two-terminal device is obtained from the from Eq.(15) by taking $s_{13} = 0$. In this case there is only one independent correlator, since $S_{11}(\omega) = S_{22}(\omega) = -S_{12}(\omega) \equiv S(\omega)$. All these correlators are different for the transport gas and the beam. Denoting $I$ as the average current in the device, one gets:

$$S^b(\omega) - S^{tr}(\omega) = eI\frac{\min(eV,\omega)}{eV}, \quad (17)$$

We now explain the classical form of the extra term in Eq.(15). This term contains transition amplitudes from states in the energy window $[\mu, \mu + min(\omega, eV)]$ to states below $\mu$ (see the second term in Eq.(9) and Eq.(10)) . Since *all* final states are empty the quantum statistics plays no role. So, with no interactions and no statistics, the particles in this energy window are independent. For *independent* particles (classical or quantum), the correlator is a sum of their single-particle correlators: $S_{\alpha\beta} = \sum_n S^{(n)}_{\alpha\beta}$ (see the second term in Eq.(9) and Ref.13). Since the classical single-particle correlator is identical to its quantum counter-part according to Eq.(14) and Ref.13, the contribution of the particles in the above energy window has a classical form.

It remains to understand why the quantum and classical single-particle correlators are equal. This is due to the averaging in Eq.(3), the unitarity of the scattering matrix and the assumption $\omega \ll \mu$. Here we will explain in detail only the role of the averaging in the vanishing of the cross-correlation: Before averaging, the single-particle temporal cross-correlator is

$$\langle\varphi_{1,k}|\hat{j}(x_2,0)\hat{j}(x_3,t)|\varphi_{1,k}\rangle = \frac{s_{21}s_{31}}{4L}e^{i\epsilon_{1,k}t}\times \\ e^{-ik(x_2-x_3)}(-i\partial_{x_2}+k)(i\partial_{x_3}+k)\langle x_2|e^{-iHt}|x_3\rangle \quad (18)$$

where $|\varphi_{1,k}\rangle \equiv \hat{a}^\dagger_{1,k}|vacuum\rangle$, and where

$$\langle x_2|e^{-iHt)}|x_3\rangle = \frac{s_{23}}{\sqrt{2\pi t}}exp(-i\frac{(x_2+x_3)^2}{2mt} + i\frac{\pi}{4}) \quad (19)$$

is the (generally nonzero) propagator from $(x_2, 0)$ to $(x_3, t)$, for $t > 0$. For simplicity, we assume the $s_{\alpha,\beta}$'s are

real and $k$-independent. The above correlator, Eq.(18), is generally different from zero (in contrast to its classical counter part). However, when applying the spatial averaging each of its terms becomes proportional to a new type of propagators: of a *wave-packet* around momentum $k > 0$ which is localized in a segment of size $L_0$ around $x_2$ on arm 2 into a similar wave-packet around $x_3$ on arm 3. This is so because the factor $e^{-ik(x_2-x_3)}\langle x_2|e^{-iHt}|x_3\rangle$ in Eq.(18) turns upon averaging into:

$$\left(\int_{L_0} \frac{dx_2}{L_0} e^{-ikx_2}\langle x_2|\right) e^{-iHt} \left(\int_{L_0} \frac{dx_3}{L_0} e^{ikx_3}|x_3\rangle\right) \quad (20)$$

where each integral is a wave-packet of the form described above. All the four terms in Eq.(18), become after averaging, proportional to propagators of similar though more complicated form. *These* propagators vanish in the limit $kL_0 \gg 1$, causing the quantum single-particle cross-correlator of the *average* current to vanish, similarly to the classical one. This vanishing has a physical meaning: if a particle is at a point $x_2$ on arm 2 it has, due to Heisenberg-principle, large momentum uncertainty and thus, although it is already on arm 2, a possibility to return and be scattered into arm 3 and reach $x_3$. However, when it is spread out in a segment of size $L_0$ around $x_2$ which is much larger than the inverse of the average momentum, its momentum uncertainty is not enough to allow it to return, and therefore, as in the classical case, it is scattered into one arm, and remains in it. Comment: without imposing the unitarity, Eq.(19) would also contain terms $\sim exp[\pm i(x_2 - x_3)^2/(2mt)]$ that would yield generally nonzero contribution also after averaging.

To conclude, using the representation of the current noise as a sum over single-particle transitions we have shown that the current correlations in time and their spectra are different[14] in a transport and a beam experiment, although the average current is the same. Thus, the picture of current in a degenerate Fermi gas as a beam of particles with energies in the transport window is grossly over-simplified. For a three-terminal device, which is a solid state analog of a beam splitting setup (from arm 1 to arms 2 and 3), the difference is given by the second term in Eq.(15), which exists only in the beam, and which start to be important at $\omega$ of order of $eV$. In the range $\omega > eV$, where there is no noise in the transport gas, this extra term gives Poissonian white noise for the auto-correlators $S_{22}(\omega)$ and $S_{33}(\omega)$, but does *not* contribute to the cross-correlator $S_{23}(\omega)$.

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[13] The single-particle classical correlator has the form Eq.(10) since a particle is scattered into arm 1 *or* 2 *or* 3 , so its cross-correlator vanishes while the auto correlation is the known Poissonian result. Formally: The classical current at a point $x_\alpha$ on arm $\alpha = 2, 3$ of a single particle incoming on arm 1 at velocity $v$ is: $a_\alpha ev\delta(x_\alpha - x_n(t))$ where $x_n(t) = -x_0 + vt$ is the particle distance from its initial point $x_0$, and where $(a_2, a_3) = (1, 0)$ or $(0, 1)$ with probabilities $T_2$ and $T_3$ respectively. Now, $S^{(n),cl}_{\alpha,\beta} \equiv \int d(t'-t)e^{i\omega(t'-t)}\langle a_\alpha a_\beta\rangle e^2v^2\langle\delta(x_\alpha - x_n(t))\delta(x_\alpha - x_n(t'))\rangle$. The first average is over scattering outcomes. The second is over $x_0$. The particle is scattered only into one arm, so, $\langle a_\alpha a_\beta\rangle = \delta_{\alpha,\beta}T_\alpha$. So, $S^{(n),cl}_{\alpha,\beta} = \delta_{\alpha,\beta}\int d(t'-t)e^{i\omega(t'-t)}I_{n,\alpha}\delta(t'-t) = e\delta_{\alpha,\beta}I_{n,\alpha}$, where. $I_{n,\alpha} \equiv evT_\alpha\langle\delta(x_\alpha - x_n(t))\rangle$ is the average (time independent) current on arm $\alpha$. For a beam of independent particles one has: $S_{\alpha\beta} = \sum_n S^{(n)}_{\alpha\beta} = e\delta_{\alpha\beta}I_\alpha$ where $I_\alpha \equiv \sum_n I_{n,\alpha}$ is the total current.

[14] Though not discussed here, the *spatial* correlations are also different in the transport gas and the beam.